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ON THE HYPOTHESIS OF ISOSTASY

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The splendid papers by Hayford[1] and jointly by Hayford and Bowie[2] have brought the subject of isostasy into the foreground for discussion by geologists and others who may be interested. These papers have taken the subject out of a field of more or less vague conjecture, and by subjecting it to a very careful quantitative examination have shown very clearly that isostasy in some form can be accepted as a reality.

To be sure, they have not *proved* the reality of isostasy, for in the mathematical sense no physical hypothesis can be proven. But they have formulated precise hypotheses of isostasy and have shown that a vast mass of observational data covering the United States is very much better satisfied by theories which include their hypotheses than by the usual gravitational theory which excludes the hypothesis of isostasy.

Four distinct laws of isostasy have been discussed in these papers, viz., (*a*) uniform compensation, (*b*) uniformly decreasing compensation, (*c*) compensation in a subterranean layer, (*d*) the Chamberlin compensation. In so far as merely satisfying the observations it is found that any one of these hypotheses is as good as any other, since each of them reduces the sum of the squares of the residuals by about 90 per cent. This is a very notable reduction and it places the hypothesis of isostasy upon a solid foundation of credibility. We say "credibility" and not "proof" advisedly, for there still remains the possibility that some other hypothesis, non-isostatic in its nature, may satisfy the observations even more closely, and it is a very difficult matter to show that no such hypotheses exist.

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[1] John F. Hayford, *The Figure of the Earth and Isostasy from Measurements in the United States*, Publications of the U.S. Coast and Geodetic Survey, 1909.

[2] John F. Hayford and William Bowie, *The Effect of Topography and Isostatic Compensation upon the Intensity of Gravity*, Publications of the U.S. Coast and Geodetic Survey, 1912.

There seems to be nothing inherently improbable in the notion that the density of the materials under the continents is less than that under the oceans. Hayford's success, which must be considered a notable one, consists in showing, by very complete computations which extend over a large mass of data, that assumptions of very moderate differences of density are sufficient to bring the observations and theory into fairly close accord. Whether or not any other hypothesis will or can be equally successful must of course be left for the future. Until some such hypothesis makes its appearance we are fairly entitled to put our faith in the broader outlines of isostasy and leave it to further observations and discussions to make the details of the theory more precise.

Notwithstanding the fact that all four of the hypotheses discussed by Hayford satisfy the observations equally well, it would seem as if Hayford prefers the hypothesis of uniform compensation to a depth of 122 kilometers and usually has this hypothesis in mind when thinking of isostasy. This preference, which does not seem to be warranted by his own discussions,[1] is somewhat dangerous in that conclusions which are peculiar to this hypothesis are given a prominence to which they are not entitled. Thus, one is rather likely to gain the impression from Hayford's writings that the "depth of compensation" is in the neighborhood of 122 kilometers and that this "depth" is as well established as are the broader outlines of the theory. This is not true, for we do not know that the compensation is uniform. From the hypothesis of uniformly decreasing compensation Hayford finds the depth of compensation to be 175 kilometers, and from the "Chamberlin compensation" 286 kilometers. Clearly, the "depth of compensation" is very sensitive to change of hypothesis, and it is further clear that with a slight modification of the hypothesis the "depth of compensation" could be made to retreat to the center of the earth, or even to vanish altogether.[1] From this it is obvious that the existence of a precise depth of compensation is not an essential part of the theory of isostasy. These considerations deprive the depth of 122 kilometers of the importance or weight which constant repetition is likely to attach to it. It is still doubtful whether the term "depth of compensation" corresponds to any physical reality, however useful the idea may be in our hypotheses.

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[1] Second, it is not possible to ascertain whether this compensation is more probable than the G compensation, uniformly distributed from the surface to the depth 70.67 miles, since the two sets of computed deflections agree so closely that their differences are much smaller than the accidental errors.—*The Figure of the Earth and Isostasy from Measurements in the United States*, p. 159.'

A corresponding statement is made on p. 162 with respect to the Chamberlin compensation.

If the solid portion of the earth were altogether lacking in rigidity, and if the concentric layers were homogeneous in density, then the upper surface of the solid earth would be an oblate spheroid, and this surface would lie about 9,000 feet below the present sea-level. It would be covered uniformly by the waters of the ocean, and the pressure at any interior point would be a function of latitude and depth only, and not a function of the longitude. Let us suppose now that this solid spheroid is endowed with a certain amount of rigidity and is differentiated somewhat with respect to density, particularly in the neighborhood of the surface. If the rigidity were not too great, it seems clear that the heavy regions would be depressed by the excessive weight, and that the lighter regions would rise on account of their deficiency of weight. If the differentiation of density were sufficiently great, it is clear that the regions of sufficiently deficient density would eventually appear above the surface of the waters of the ocean, while the regions of greatest density would form the great depths of the basins into which the the waters of the ocean were gathered. Those regions which have neither excess nor deficiency of density would still remain about 9,000 feet below the sea-level except in so far as, through rigidity, they partook of the movement of neighboring regions. A theory of isostasy comprehending the entire earth in its grasp should therefore be based upon a level 9,000 feet below the surface of the ocean. Regions which lie above this level are deficient in density and regions which lie below this level are excessive in density. Clearly, it would not do to regard the sea-level, which from an isostatic point of view is an accidental level, as the dividing surface. Indeed, if the differentiation of density were so small that the elevated regions were all below the sea-level, we should be compelled to conclude that the density was everywhere excessive—a *reductio ad absurdum.*

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[1] The idea implied in this definition of the phrase "depth of compensation" that the isostatic compensation is complete within some depth much less than the radius of the earth is not ordinarily expressed in the literature of the subject, but it is an idea which is difficult to avoid if the subject is studied carefully from any point of view.—Hayford and Bowie, *The Effect of Topography and Isostatic Compensation upon the Intensity of Gravity*, p. 10.

But the sea-level is precisely the surface which Hayford has chosen, and, as a consequence, to all those regions which lie between the sea-level and 9,000 feet thereunder he has ascribed an excess of density instead of a deficiency. Since approximately three-quarters of the earth's surface lies under the ocean at an average depth of approximately 15,000 feet, and since under Hayford's hypotheses too great density is ascribed to this region, it would seem that his hypothesis has had the effect of raising the mean surface density for the entire earth from 2.67 to a somewhat higher figure. This alone might not be of any great importance, since at best the figure 2.67 is somewhat uncertain, but a hypothesis which is based upon sea-level gives horizontal changes of defect of density over the continents which are relatively too great, as is shown in Table I, which is based upon the hypothesis of uniform compensation.

Thus, with the sea-level basis the defect in density under a region which has an altitude of 6,000 feet is six times as great as the defect in density under a region which has an elevation of 1,000 feet, while with a true, isostatic theory the defect would be only one and one-half times as much. The defect under a 1,000-foot elevation is twice as great as under a 500-foot elevation on the one hypothesis and only twenty-nineteenths times as great on the other. This indicates that the sea-level basis overaccentuates the importance of changes of level in the topography of the continents.

TABLE I

Elevation in Feet	Defect in Density Sea-Level Basis	Defect in Density 9,000-Foot Basis	Elevation in Feet	Defect in Density Sea-Level Basis	Defect in Density 9,000-Foot Basis
0.......	0	9	4,000......	4	13
1,000.......	1	10	5,000......	5	14
2,000.......	2	11	6,000......	6	15
3,000.......	3	12	7,000	7	16

If the density in the earth's crust actually varies in the manner supposed by Hayford (using the sea-level basis), one would expect regions over which the compensation was effected to be smaller than if the density varies according to a true isostasy (i.e., the 9,000-feet-below-the-sea basis), since the changes in density are relatively greater in the first case than in the second. Hayford has attempted to determine the sizes of these areas of compensation, but the quantities to be considered were so small that success was scarcely to be expected; indeed, they are "frequently less than the errors of observation and computation" and, possibly, also less than the effects of local irregularities of density. The evidence, though inconclusive, leaned faintly toward rather small areas of compensation, and Hayford is of the opinion that these areas are between a square mile and a square degree.[1] If the present writer is correct in assuming that a true isostasy must be based upon a level 9,000 feet below the sea, then the evidence published by

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[1] It is certain from the results of this investigation that the continent as a whole is closely compensated, and that areas as large as states are also closely compensated. It is the writer's belief that each area as large as one square degree is generally largely compensated. The writer predicts that future investigations will show that the maximum horizontal extent which a topographic feature may have and still escape compensation is between one square mile and one square degree. This prediction is based, in part, upon a consideration of the mechanics of the problem.—Hayford and Bowie, *The Effects of Topography and Isostatic Compensation upon the Intensity of Gravity*, p. 102.

Hayford and Bowie on the extremely delicate question[1] of the "areas of compensation" is not altogether trustworthy, and the sizes of these areas must be regarded as unknown.

From the fact that the hypothesis of isostasy reduced the sum of the residuals from 65,434 to 8,013, or by approximately 90 per cent, and from the fact that the average elevation of the United States is about 2,500 feet Hayford concluded that the average departure from complete isostasy in the United States is equal to about 250 feet of rocks. It is not easy to see how Hayford drew this conclusion. It certainly has no mathematical justification, for even if the theory were perfect and the isostasy complete, the sum of the squares of the residuals would not be zero, since the imperfections of the observations would still give us a very respectable, but quite unknown, sum. How then can we form a quantitative judgment from the sum of the squares of the residuals which depends, not only upon the imperfections of the theory, but upon the imperfections of the observations as well? Obviously, it cannot be done. The estimate of 250 feet is little more than a guess, however shrewd the guess may be. If we use the 9,000-foot level as the basis of our guess, then the average elevation of the United States is 11,500 feet, and the average departure from complete isostasy is 1,150 feet of rocks instead of 250 feet. It would not be altogether fair, however, to make this direct substitution, for the reduction of the sum of the squares of the residuals from 65,434 to 8,013 was accomplished on the sea-level hypothesis, and the reduction might be quite different under another hypothesis.

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[1] If the separate anomalies in the United States be compared, it is found that in 16 cases out of 41 the anomaly with local compensation assumed is smaller than with regional compensation assumed uniformly distributed to zone K (18.8 kilometers), and only 13 cases in which it is larger. Similarly, there are 20 cases out of 41 in which the anomaly with local compensation is smaller than with regional compensation extending to zone M (58.8 kilometers), and only 15 cases in which it is larger. There are 26 cases out of 41 in which the anomaly with local compensation assumed is smaller than with regional compensation assumed to extend to zone O (166.7 kilometers), and only 12 cases in which it is larger. In all other cases the two anomalies compared are identical to the last decimal place used, the third.

The evidence either for or against local compensation in comparison with such regional compensation distributed uniformly over these moderate distances is necessarily slight and possibly inconclusive. For, as shown in the table, the difference between computed effects of compensation in the two cases compared is very small upon an average. The whole evidence is furnished by these very small differences, which frequently are less than the errors of observation and computation. As shown by the table, there is but one station among the 41—namely, No. 43, Pike's Peak—at which the difference between the computed effect of local compensation and the computed effect of regional compensation uniformly distributed to zone K exceeds 0.004. Such a difference tends to become greater as the distance over which the regional compensation is supposed to be uniformly distributed is increased, but columns 7 and 8 of the table show that even when the regional compensation is assumed to extend to zone O, a distance of 166.7 kilometers, from the station, there is only one station among the 41—namely, station no. 54, San Francisco—at which the computed effect of local compensation and the computed effect of regional compensation exceeds 0.017 dyne.

Nevertheless the evidence, slight as it necessarily is, indicates that the assumption of local compensation is nearer the truth than the assumption of regional compensation uniformly distributed to zone K (18.8 kilometers). The evidence is still stronger in the same direction when the comparison is made between local compensation and regional compensation extending uniformly to the greater distances, 58.8 and 166.7 kilometers, represented by zones M and O.—Hayford and Bowie, *The Effects of Topography and Isostatic Compensation upon the Intensity of Gravity*, p. 101.

From a purely mathematical point of view, any set of a finite number of observations of the intensity and direction of gravity can be satisfied, not approximately, but exactly, in infinitely many ways by a proper distribution of density in the earth. The virtue of the theory of isostasy, therefore, lies, not in the mere fact that the observations are more nearly satisfied by the theory than without it, but in the fact that a definite principle is laid down for the variations of density, and that this principle brings theory and observations into a satisfactory accord. As Hayford's four distinct hypotheses show, any smoothly uniform hypothesis of isostasy can be regarded only as a first approximation to the actual situation, and Hayford has been successful in showing that any one of these four hypotheses is a good first approximation. It is equally clear that such delicate points as "depth of compensation" and size of "areas of compensation" depend for their successful determination upon the vastly more difficult matter of second and higher approximations, and these approximations can be obtained, if at all, only by a very much more dense net of observations, and quite likely the observations themselves would have to be still further refined.

While the theory of isostasy has made a very successful approach to the solution of the problem of bringing the anomalies of observation into accord with the theory of gravity, it must be admitted that there is no evidence to show that the solution of the problem is necessarily isostatic.